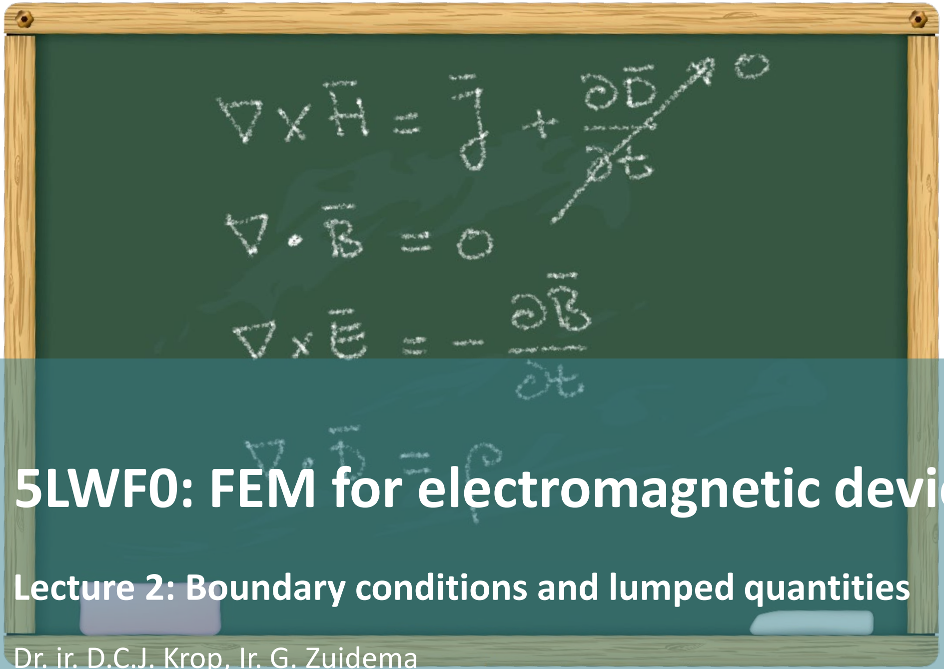

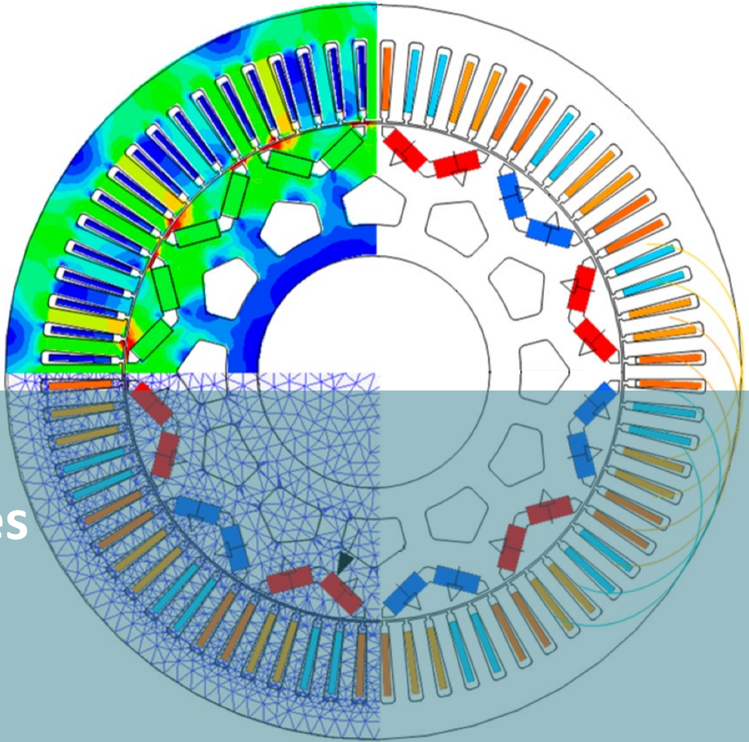

5LWF0: FEM for electromagnetic devices

Lecture 2: Boundary conditions and lumped quantities

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Outline for the lecture of week 2

Recognizing and exploiting boundary conditions

- Truncation of problem space through boundary conditions
- Model reduction by exploiting geometric and physical symmetries and/or periodicities

EMF: Electromotive force

- Transformer EMF (induced by time-variation of *stationary* time-varying sources)
- Movement EMF (induced by *movement* of time-(in)variant sources)

Inductances and materials

- Self- and mutual inductance calculations for non-linear magnetic materials
- Different interpretation/results for different solver types

Interpreting results from different formulations of the PDE (solver types)

- Comparison of results for the same problem solved with different formulations of the PDE

Recognizing and exploiting boundary conditions

Boundary conditions in electromagnetic models

Selection of the solver-type in FLUX2D

- Sets the formulation of the PDE
- Order of the mesh elements determines the shape functions
- The stiffness matrix depends on the **discretized geometry (*x*,*y*)** and **material properties (ν)** and therefore determines the solution

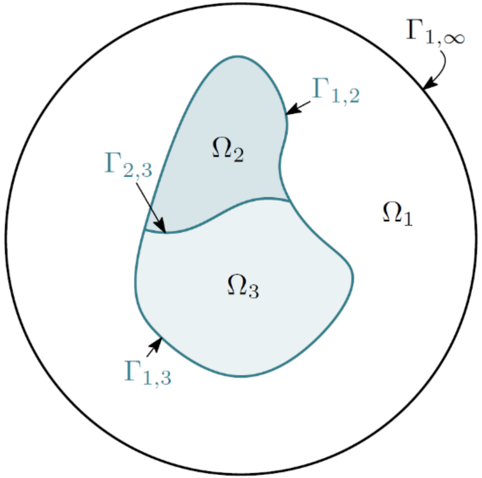
Each ***line*** in a 2D-problem is interpreted as a boundary (of a domain) → drawing lines is basically defining the ***position*** of the boundary (of a domain)

- Space itself has electromagnetic properties → truncation of the domain necessary to obtain a **finite** number of elements for computation

Boundary interpretation in Flux models

When ***not*** actively assigned

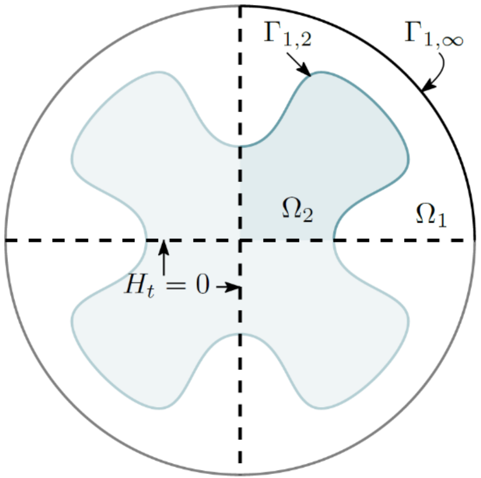
- General magnetic boundary conditions apply between ***internal domains*** with the surface current density set to zero
- Normal magnetic field BC automatically, i.e. a *natural BC*, applies on the ***outer contour*** enclosing the problem space not being of the infinite box type



Region with a surface current density, J_Γ, on the line into or out of the plane

- *Discontinuous* tangential *H*-field component in the domains on either side of the line

Boundary interpretation in Flux models



Normal magnetic field (Neumann)

- Tangential component of H-&B-field are forced to be zero (to exploit symmetries)

Tangential magnetic field (Neumann)

- Normal component of H-&B-field are forced to be zero (to exploit symmetries)

Imposed magnetic flux or vector potential value (Dirichlet)

- Directly assigning value of the state-variable, A_z, to the boundary
- When set to a constant value, essentially the same as tangential magnetic field BC (*Why?*)

Model reduction by assigning special boundary conditions

How to recognize symmetry in problems and how to apply the proper BCs?

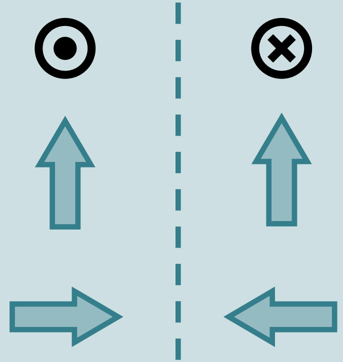
- Search for **geometrical** symmetry first
- Check if this symmetry also holds w.r.t. **polarities of sources**

Only tangential field component along axis of symmetry

$$J_{\mathrm{L}} = -J_{\mathrm{R}}$$

$$M_{\mathrm{L}\parallel} = M_{\mathrm{R}\parallel}$$

$$M_{\mathrm{L}\perp} = -M_{\mathrm{R}\perp}$$

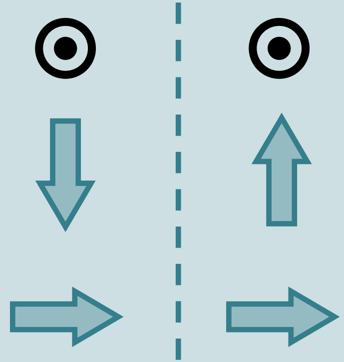


Only normal field component along axis of symmetry

$$J_{\mathrm{L}} = J_{\mathrm{R}}$$

$$M_{\mathrm{L}\parallel} = -M_{\mathrm{R}\parallel}$$

$$M_{\mathrm{L}\perp} = M_{\mathrm{R}\perp}$$



Model reduction by assigning special boundary conditions

How to recognize periodicity in problems and how to apply the proper BCs?

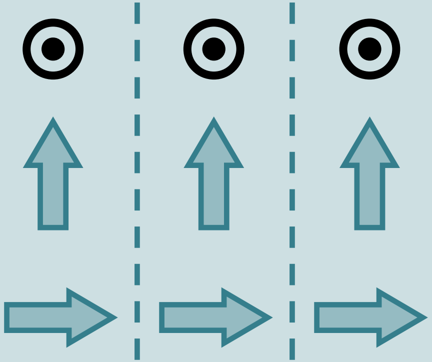
- Search for **geometrical** periodicity (if so, distance/angle denoted by τ)
- Check if this (anti-)periodicity also holds w.r.t. **polarities of sources**

A_z equal in *<u>both</u>* magnitude *<u>and</u>* polarity on axes of periodicity

$$J(\vec{x}) = J(\vec{x} + k\tau)$$

$$\vec{M}(\vec{x}) = \vec{M}(\vec{x} + k\tau)$$

$$\forall k \in \mathbb{Z}$$

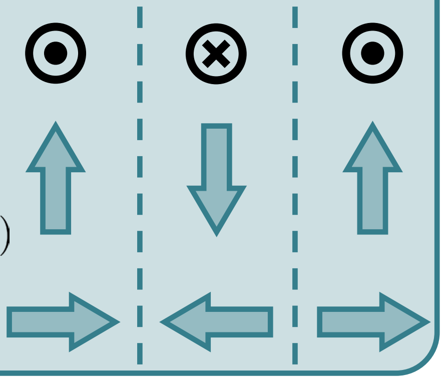


A_z equal in magnitude, but *<u>opposite</u>* polarity on axes of anti-periodicity

$$J(\vec{x}) = (-1)^k J(\vec{x} + k\tau)$$

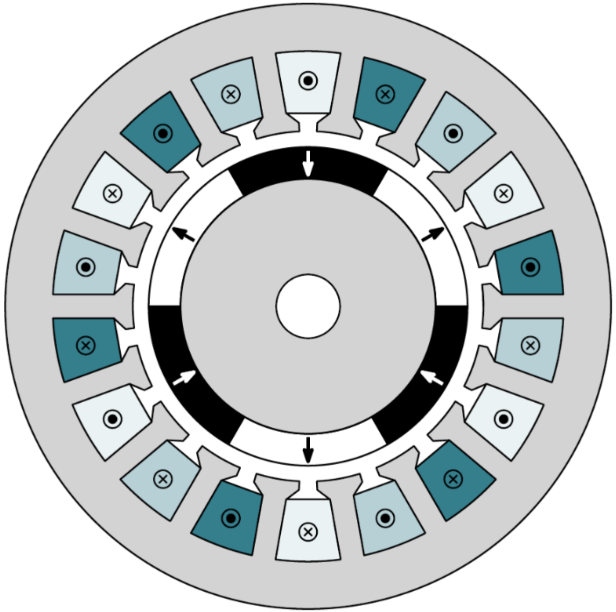
$$\vec{M}(\vec{x}) = (-1)^k \vec{M}(\vec{x} + k\tau)$$

$$\forall k \in \mathbb{Z}$$



Model reduction example with (anti-)periodicity

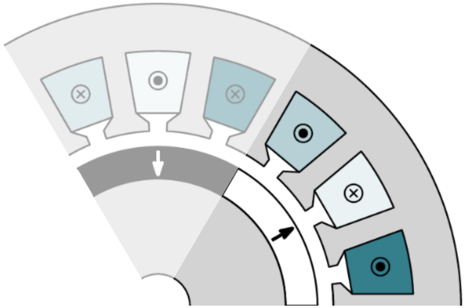
Full model



Anti-periodicity of $\tau = 60°$

Periodicity of $\tau = 120°$

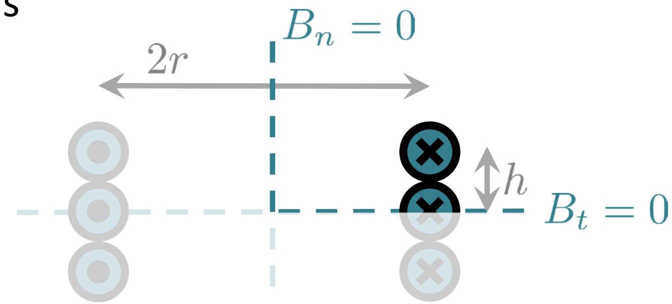
Reduced model



Model reduction example with symmetry

Coil in *free* space with 2*N*+1 turns

$$B_0 = \frac{\mu_0 I}{2\pi}$$



Analytical solution in the *xy*-plane (imaging)

$$B_x = \sum_{n=-N}^{N} \frac{B_0(y-nh)}{(x-r)^2+(y-nh)^2} + \frac{-B_0(y-nh)}{(x+r)^2+(y-nh)^2}$$

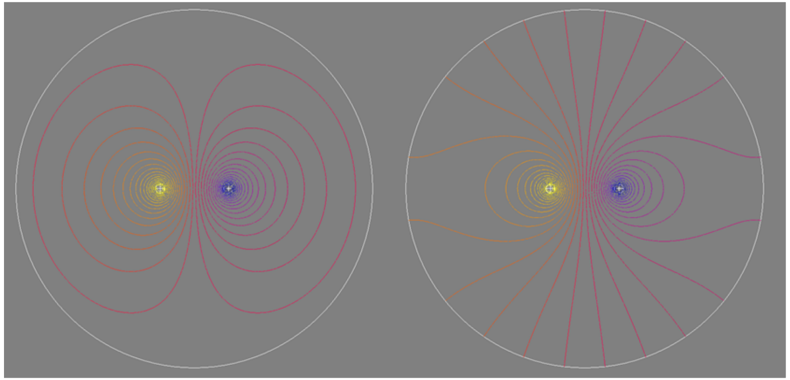
$$B_y = \sum_{n=-N}^{N} \frac{-B_0(x-r)}{(x-r)^2+(y-nh)^2} + \frac{B_0(x+r)}{(x+r)^2+(y-nh)^2}$$

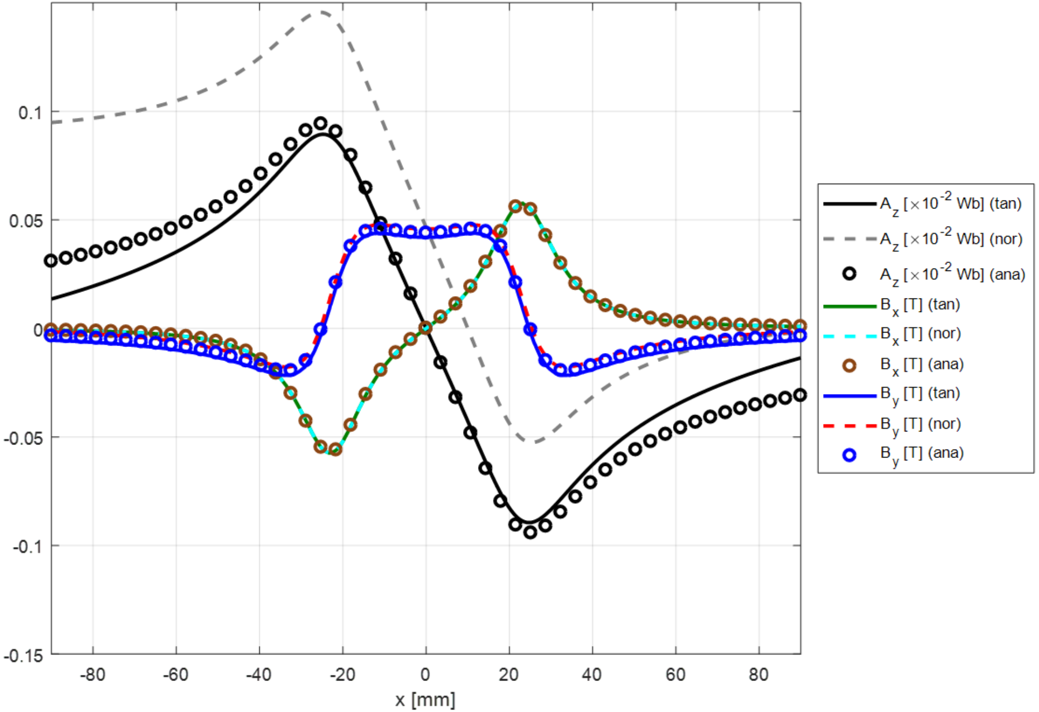
Effect of domain truncation: circular boundary

Field values along a line @ -90 ≤ x ≤ 90; y = 10 and N=0

tangential BC / fixed potential:
B_n=0 / A=constant

normal BC:
B_t=0





Effect of domain truncation: rectangular boundary

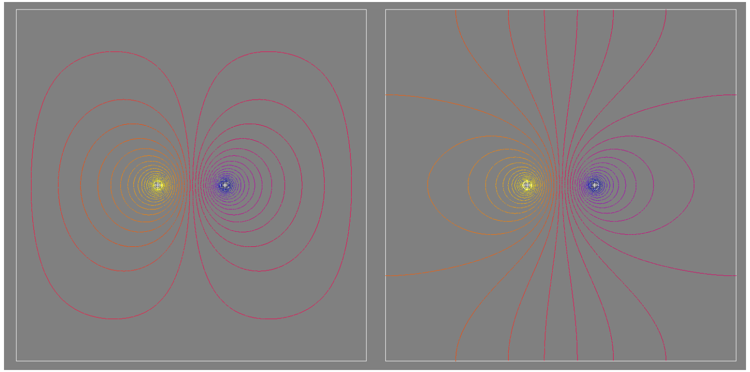
Field values along a line @ -90 ≤ *x* ≤ 90; *y* = 10 and *N*=0

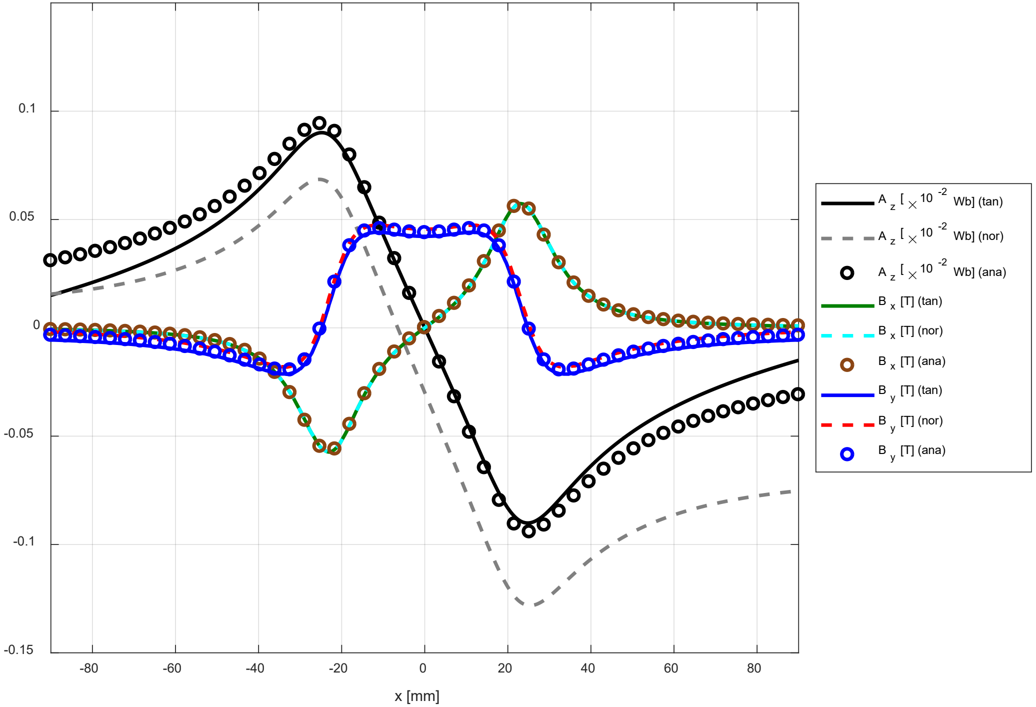
<u>tan</u>gential BC / fixed potential:

B_n=0 / A=constant

<u>nor</u>mal BC:

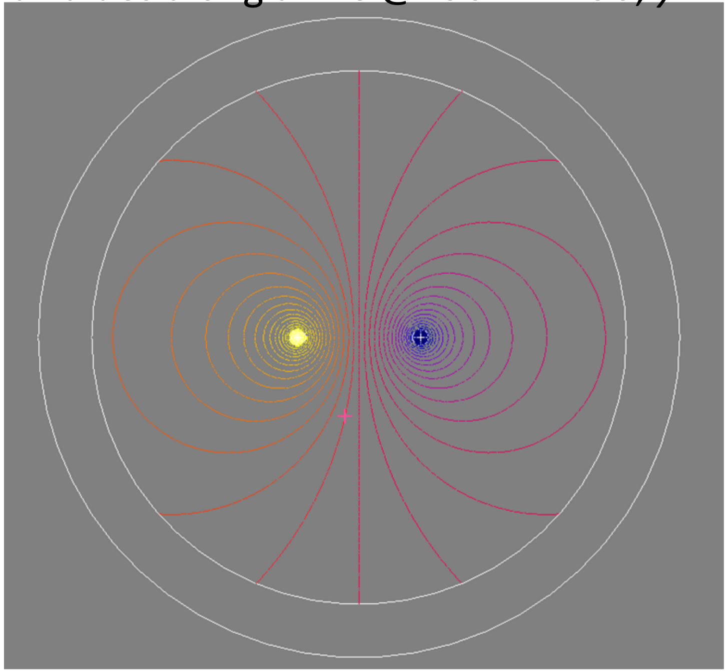
B_t=0

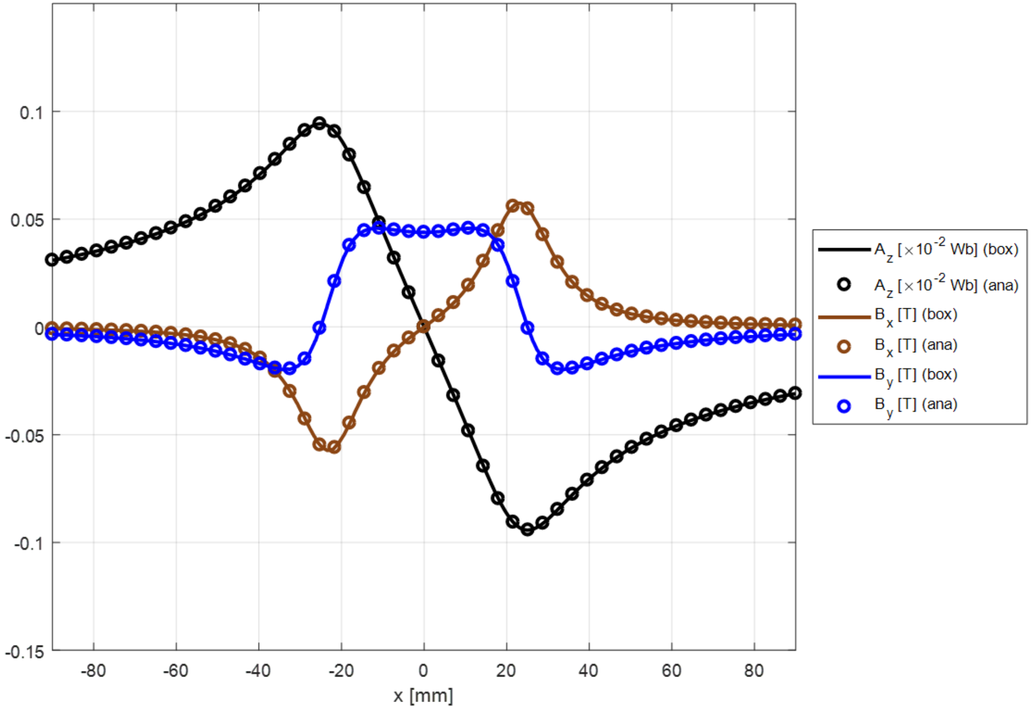




Effect of domain truncation: infinite box

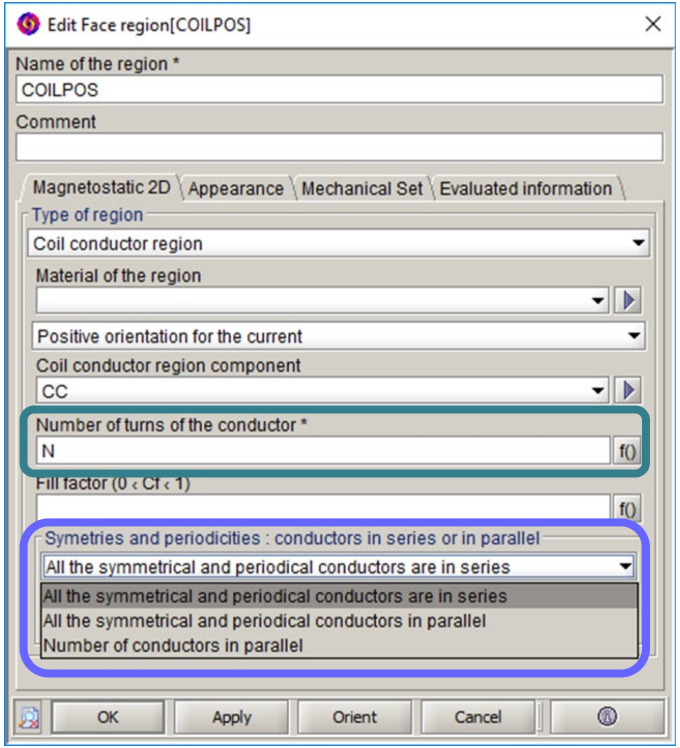
Field values along a line @ $-90 \le x \le 90$; $y = 10$ and N=0





Outline	**Boundary conditions**	Electromotive force	Inductances & materials	Formulations of the PDE

Assigning boundary conditions in Flux



All in series:

- MMF in ***each individual*** source region equal to the assigned values of current times **number of turns**

All in parallel:

- Total MMF over ***all*** source regions equal to the assigned values of current times **number of turns**

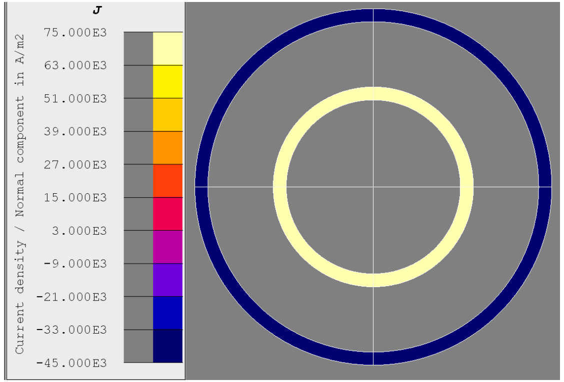
Number of conductors in parallel

- Total MMF over ***all*** source regions equal to the assigned values of current times **number of turns** divided by the **number of conductors in parallel**

Assigning boundary conditions in Flux

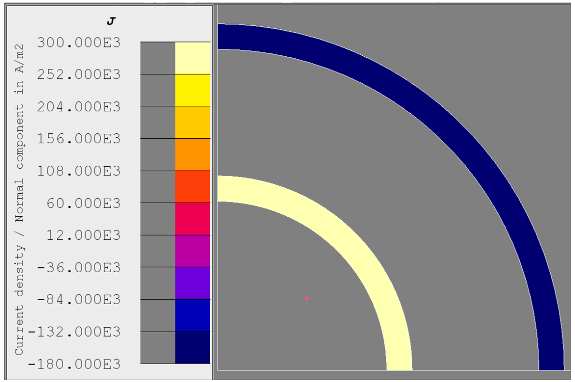
Full model:

-45 kA $\leq J \leq 75$ kA



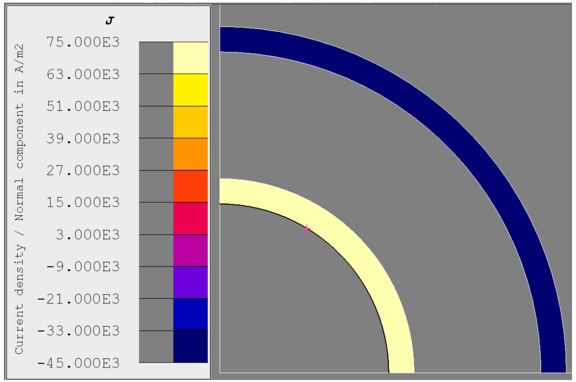
All in series:

-180 kA $\leq J \leq 300$ kA



All in parallel:

-45 kA $\leq J \leq 75$ kA

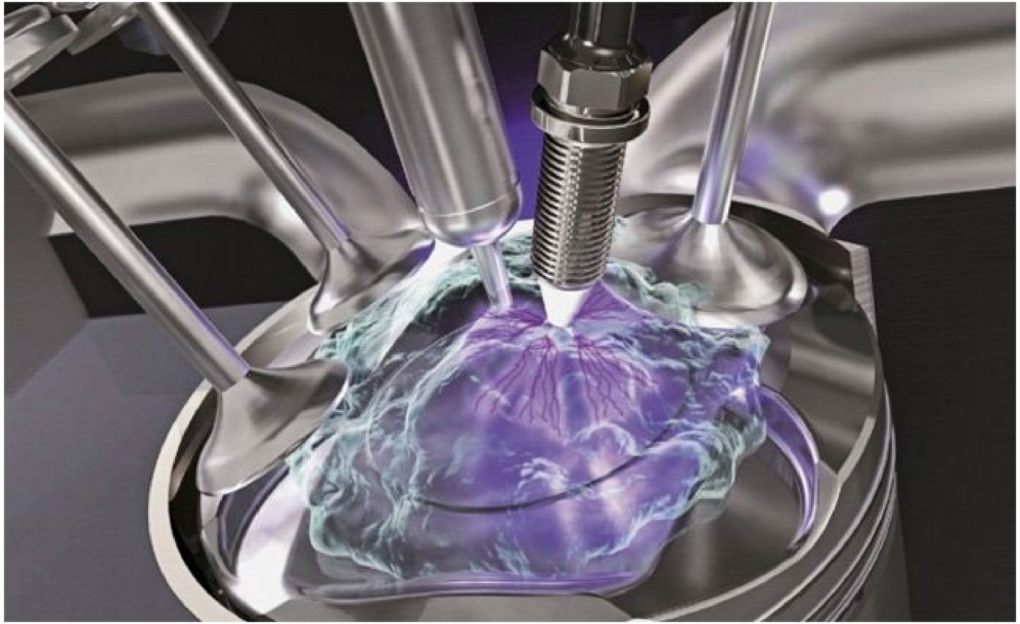


Assigning boundary conditions in Flux

FLUX WILL AWAYS PROVIDE A SOLUTION SUBJECT TO THE ASSIGNED BOUNDARY CONDITIONS!

HOWEVER, THE SOLUTION WILL BE PHYSICALLY MEANINGLESS WHEN THE WRONG BOUNDARY CONDITIONS ARE APPLIED!

EMF: ElectroMotive Force



From spatial fields to lumped circuit components

Analysis of electromechanical **systems** is mainly conducted on an electric circuit level, rather than a spatial field distribution level

Electrical behavior in a system

- Function of the mechanical load
- Electrical behavior in combination with a power electronic drive

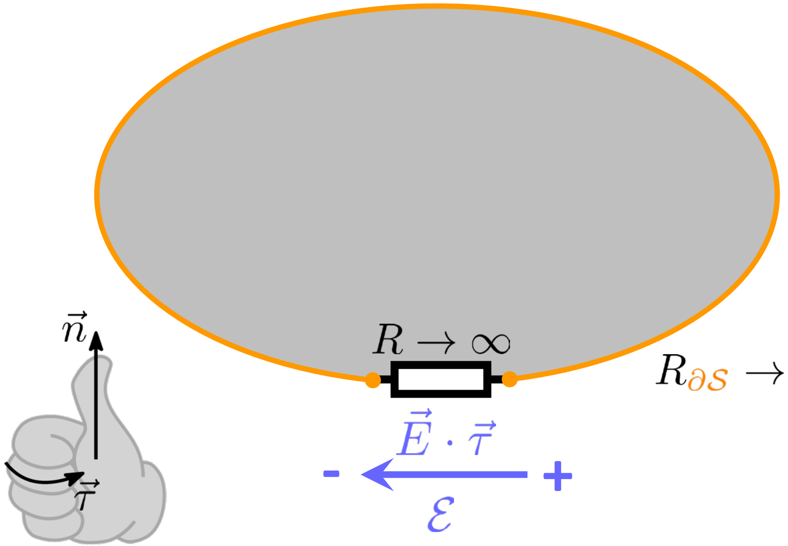
Circuit representation through lumped circuit components derived form *global* (non-spatial) field quantities

- **Active** source component
  - Voltage/EMF sources as function of mechanical and other electric quantities
- **Passive** components: *R*, *L* (and *C*)
  - Reduction from spatial field quantities relations to simple voltage-current relations

Active, lumped circuit component: EMF source

The ElectroMotive Force (**EMF**) originates from Faraday's law:

- A **time varying flux linkage** through a **surface** that is enclosed by a **contour** results in a **time varying voltage** over/along the **contour** (Polarity via Lenz's law result of the right-hand-rule)



$$R_{\partial S} \to 0 : E_\tau \to 0$$

$$\oint_{\partial S} \vec{E}(\vec{r},t) \cdot \vec{\tau}\,\mathrm{d}\ell = -\frac{\mathrm{d}}{\mathrm{d}t} \iint_S \vec{B}(\vec{r},t) \cdot \vec{n}\,\mathrm{d}S$$

$$= -\frac{\mathrm{d}}{\mathrm{d}t} \iint_S \left(\nabla \times \vec{A}(\vec{r},t)\right) \cdot \vec{n}\,\mathrm{d}S$$

Stokes

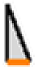
$$= -\frac{\mathrm{d}}{\mathrm{d}t} \oint_{\partial S} \vec{A}(\vec{r},t) \cdot \vec{\tau}\,\mathrm{d}\ell$$

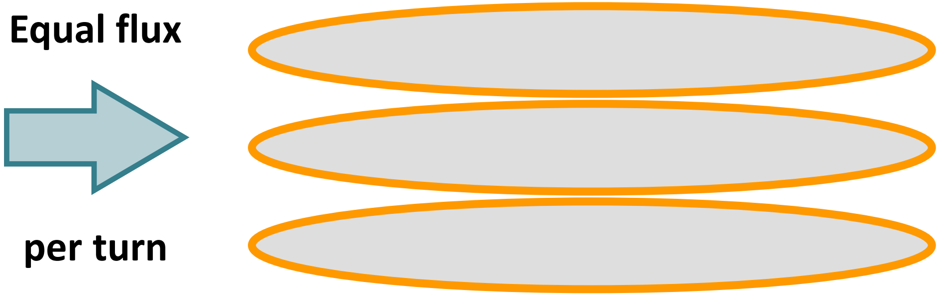
$$\mathcal{E} = -\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

The global quantity flux linkage

The flux linkage of an N-turn coil:

- A single complex surface that is still enclosed by a single contour



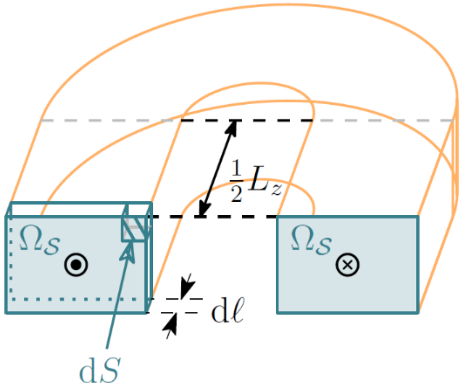


$$\Psi = \sum_{n=1}^{N} \phi_n = \oint_{\partial S} \vec{A}(\vec{r}, t) \cdot \vec{\tau} \, \mathrm{d}\ell$$

$$\Psi = N\phi \;\Rightarrow\; \phi = \frac{1}{N} \oint_{\partial S} \vec{A}(\vec{r}, t) \cdot \vec{\tau} \, \mathrm{d}\ell$$

The global quantity flux linkage

The flux linkage of a physical N-turn coil: non-infinitesimal conductor cross-section



- Consider the volume of the coil to be a single turn
- The flux seen by the turn equals the **average** flux in the volume

$$\phi = \frac{1}{\mathcal{S}} \iint_{\Omega_S} \left[\oint_{\mathcal{C}} \vec{A} \cdot \vec{\tau}\, \mathrm{d}\ell \right] \mathrm{d}S = \frac{1}{\mathcal{S}} \oint_{\mathcal{C}} \iint_{\mathcal{S}} \vec{A} \cdot \vec{\tau}\, \underbrace{\mathrm{d}S \mathrm{d}\ell}_{\mathrm{d}V} = \frac{1}{\mathcal{S}} \iiint_{\mathcal{V}} \vec{A}\, \mathrm{d}V$$

$$\Psi = \frac{N}{\mathcal{S}} \iiint_{\mathcal{V}} \vec{A}\, \mathrm{d}V$$

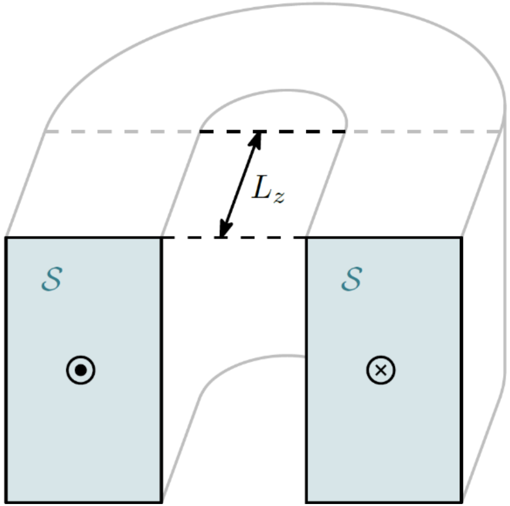
The global quantity flux linkage

The flux linkage of an inductor is a **global** quantity that represents the total amount of **flux** seen by the inductor, multiplied by the number of **turns**

$$\Psi = N\phi$$

Flux equals the average amount of MVP in its volume, i.e. 2D:

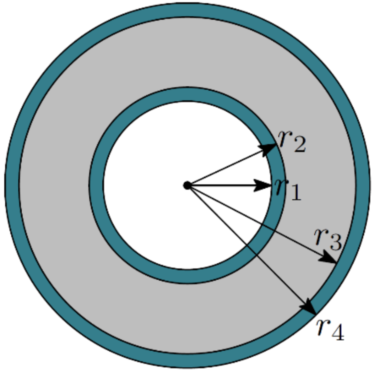
$$\phi = \frac{L_z}{a} \iint_{\Omega_S} A_z \,\mathrm{d}S \text{ where: } a = \iint_{\Omega_S} 1 \,\mathrm{d}S$$

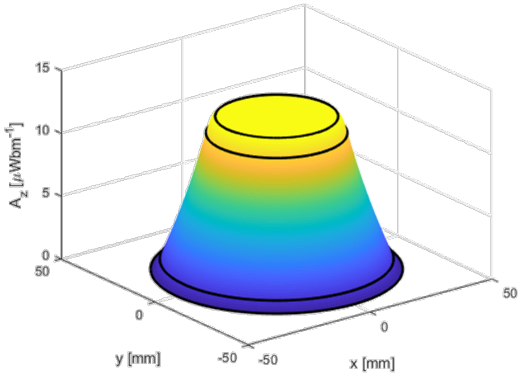


Hence (in Cartesian coordinates)

$$\Psi = \frac{NL_z}{a} \iint_{\Omega_S} A_z(x,y) \,\mathrm{d}x\mathrm{d}y$$

Example from instruction 1: the toroidal inductor





MVP expression: $A_k(r) = A_{0,k} + B_{0,k} \ln r - \frac{1}{4} J_{0,k} r^2$

flux linkage: $\Psi = \dfrac{N L_z}{S} \iint_S A_z(x,y) \mathrm{d}S$

region surface area: $S_k = \pi \left(r_k^2 - r_{k-1}^2 \right)$

$$\Psi = \frac{N L_z}{\pi \left(r_2^2 - r_1^2 \right)} \int_0^{2\pi} \int_{r_1}^{r_2} \left[A_{0,2} + B_{0,2} \ln r - \tfrac{1}{4} J_{0,2} r^2 \right] r \mathrm{d}r \mathrm{d}\theta$$

$$- \frac{N L_z}{\pi \left(r_4^2 - r_3^2 \right)} \int_0^{2\pi} \int_{r_3}^{r_4} \left[A_{0,4} + B_{0,4} \ln r - \tfrac{1}{4} J_{0,4} r^2 \right] r \mathrm{d}r \mathrm{d}\theta$$

Return path of the coil: $\mathrm{d}\vec{\ell} = \mathrm{d}z < 0$

Nature of EMF inducing time-varying flux linkage

Two (artificial) ways of time variations

- Time-varying currents/fields through stationary coils: transformer EMF
- Relative movement of coils in a stationary magnetic field: EMF due to relative movement

$$\mathcal{E} = -\frac{\mathrm{d}\Psi}{\mathrm{d}t} \qquad = -\frac{\mathrm{d}LI}{\mathrm{d}t}$$

$$\mathcal{E} = -\frac{\mathrm{d}\Psi}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}\theta} \qquad = -\left(L\frac{\mathrm{d}I}{\mathrm{d}t} + I\frac{\mathrm{d}L}{\mathrm{d}t}\right)\frac{\mathrm{d}\theta}{\mathrm{d}\theta}$$

$$\mathcal{E} = -\frac{\mathrm{d}\Psi}{\mathrm{d}\theta}\omega_{\mathrm{mech}} \qquad = -\left(L\frac{\mathrm{d}I}{\mathrm{d}\theta} + I\frac{\mathrm{d}L}{\mathrm{d}\theta}\right)\omega_{\mathrm{mech}}$$

Is an EMF induced by moving permanent magnets ***exclusively*** *the result of 'EMF due to relative movement'?*